MIE100: Lecture starts at 10 minutes after the hour

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Lecture slides can be downloaded for each lecture from the Quercus module:

"Instructor Section: Anthony Sinclair"

If you find an error in my lecture slides, or have suggestions on how a point could be made more clear, please e-mail me at sinclair@mie.utoronto.ca

MIE 100 : Lecture #2

Meriam 2/2 : Rectilinear Motion

In our last lecture, we looked at kinematic relationships among : s, v, a, t

$$v = \frac{ds}{dt} \quad ; \quad a = \frac{dv}{dt}$$

We want to develop expressions for v, s in cases where :

(i) $a = a(t)$

$$\rightarrow \quad v = v_0 + \int_0^t a(t')\, dt'$$

$$s = s_0 + v_0 t + \int_0^t \left[\int_0^{t'} a(t'')\, dt'' \right] dt'$$

(ii) $a = a(s)$

(iii) $a = a(v)$

(ii) $a = a(s)$

Eg:

$$v = \frac{ds}{dt} \rightarrow dt = \frac{ds}{v}$$

$$\& \quad a = \frac{dv}{dt} \rightarrow dt = \frac{dv}{a}$$

combine

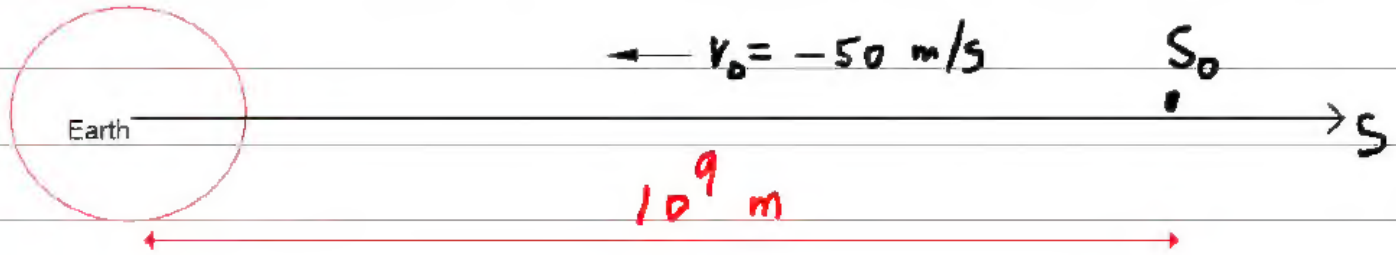
$$\rightarrow \frac{ds}{v} = \frac{dv}{a} \rightarrow \boxed{v\,dv = a\,ds}$$

$$\int_{s_0}^{s} a(s')\,ds' = \int_{v_0}^{v} v'\,dv' = \tfrac{1}{2}\left(v^2 - v_0^2\right)$$

$$\rightarrow v^2 = v_0^2 + 2\int_{s_0}^{s} a(s')\,ds' \rightarrow v(s) \ *$$

if $a(s) = a_0$ $\rightarrow v_{final}^2 = v_0^2 + 2a_0(s_{final} - s_0)$

Eg: Find impact speed of an asteroid falling to earth from an altitude of 10^9 m (Ignore air resistance.) $V_0 = -50$ m/s.



$$\int_{S_0}^{S_{final}} a\,ds = \int_{V_0}^{V_{final}} v\,dv = \tfrac{1}{2}\left(V_{final}^2 - V_0^2\right)$$

$a \rightarrow F_g/m$

Use S.I. units

$$\int_{10^9}^{6.4E6} -\left(\frac{G\,M_{earth}\,m}{s^2}\Big/ m\right) ds = \tfrac{1}{2}\left(V_{final}^2 - (-50)^2\right)$$

$$G\,M_{earth}\left(\frac{1}{S_{final}} - \frac{1}{S_0}\right) = \frac{1}{2}\left(V_{final}^2 - 2500\right)$$

$G = 6.67E-11$
$M_{earth} = 6E24$
} S.I. units

$\rightarrow$ $V_{final} = +/- 11{,}200$

$\rightarrow$ $\boxed{V_{final} = -11{,}200 \text{ m/s}}$

Also, $\frac{ds}{dt} = v(s)$

$$\int_{s_0}^{s} \frac{ds'}{v(s')} = \int_{0}^{t} dt' = t$$

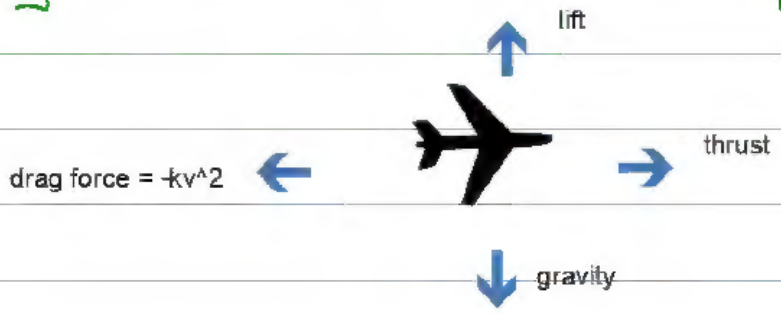
where we get $v(s)$ from:

$$v(s) = \sqrt{v_0^2 + 2\int_{s_0}^{s} a(s')ds'}$$ ✳

→ In theory, I can find s for any value of t, but arithmetic looks messy

(iii) $a = a(v)$

Eg. Drag force from air or a liquid



Drag Force on an airplane

- **Lift-induced drag**
- **Parasitic drag**
  - skin friction
  - form (shape) drag
  - airflow interference

$$* \quad a(v) = \frac{dv}{dt} \rightarrow \int_{V_0}^{V} \frac{dv'}{a(v')} = \int_0^t dt' = t \rightarrow v(t)$$

and

$$** \quad v(t) = \frac{ds}{dt} \rightarrow \int_0^t v(t')dt' = \int_{S_0=0}^{S} ds' = \Delta s = s(t)$$

Eg. A projectile travels through a fluid:

$V_0 = 60$ m/s

$a = -(0.4 v^3)$ m/s^2 (S.I. units)

Find v, x after 4 seconds

Answer: From *,

$$\int_{60 m/s}^{V} \frac{dv'}{-0.4(v')^3} = t$$

$$\rightarrow \left(\frac{-1}{0.4}\right)\left(\frac{-1}{2}\right)\frac{1}{(v')^2}\Big|_{60}^{V} = \frac{1}{0.8}\left[\frac{1}{v^2} - \frac{1}{60^2}\right] = t$$

$$\rightarrow \boxed{v = \left[\frac{1}{60^2} + 0.8t\right]^{-1/2} \text{ m/s}}$$

at $t = 4$ seconds, $v = 0.559$ m/s

From **, $x(t=4s) = \int_0^{t=4} \left(\frac{1}{60^2} + 0.8t'\right)^{-1/2} dt'$

$$x(4) = \frac{2}{0.8}\left(\frac{1}{60^2} + 0.8t'\right)^{1/2}\Big|_0^{t=4} = \boxed{4.43 \text{ m}}$$

Unit Conversions

Eg: Convert 80 km/hr to S.I. units.

Answer:

$$80\ \frac{km}{hr} \times \frac{1\ hr}{3600\ s} \times \frac{1000\ m}{km}$$

$\frac{1\ hr}{3600\ s} = 1$ $\quad$ $\frac{1000\ m}{km} = 1$

$$\frac{80}{3.6}\ m/s = 22.2\ m/s$$

Unit Balance

$$v^2 = v_0^2 + 2a_0 \Delta s$$

m^2/s^2 $\quad$ m^2/s^2 $\quad$ $(m/s^2)(m)$

Summary

We have looked at kinematic eqs for rectilinear motion:

Given a_o, $a(t)$, $a(s)$, $a(v)$ → Find v, s

1/ We found that the case of acceleration = a_o, $a(t)$ } fairly straight forward

2/ For $a(s)$, $a(v)$ → nastier arithmetic

why?

For next lecture:

Read Meriam 2/3, 2/4

Plane Curvilinear Motion